Commutative Algebra and Algebraic Geometry
CAAG 2016

11 – 15 October, 2016

Schedule and Abstracts of Talks

Indian Institute of Science Education and Research (IISER)
Mohali

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Umesh Dubey	*Thick subcategories of singularity category*
Sudhir Ghorpade	*Hyperplane sections of Schubert varieties over finite fields*
Dipankar Ghosh	*Characterizations of regular local rings via syzygy modules of the residue field*
Rajendra Gurjar	*A simplified proof of a theorem of M. Miyanishi*
Krishna Hanumanthu	*Positivity of line bundles on blow ups of the complex projective plane*
Ananthnarayan Hariharan	*Associated Graded Rings and Connected Sums*
Amit Hogadi	*Gabber's presentation lemma for finite fields*
Sagar Kolte	*Injectivity of the Vaserstein Symbol*
Rajiv Kumar	*Herzog-Kuhl Equations and its Applications*
Jai Laxmi	*Decomposing Gorenstein Rings as Connected Sums*
Yashonidhi Pandey	*Brauer group of the moduli stack and space of parahoric torsors*
Tony Joseph Puthenpurakal	*Symmetries and connected components of the AR-quiver*
Rakesh Reddy	*Relative Hilbert Coefficients*
Selvaraja S	*Regularity of powers of bipartite graphs*
Joydip Saha	*Primary decomposition and Betti numbers of ideals of the form $I_1(XY)$*
Parangama Sarkar	*Local cohomology of multi-Rees algebras, joint reduction numbers and product of complete ideals*
Husney Parvez Sarwar	*K-theory of monoid algebras*
Jyoti Singh	*Generalized Eulerian property of graded local cohomology modules*
Prasant Singh	*Maximal linear sections of 2-step flag varieties and quadratic Veronese of Grassmannians*
Gaurab Tripathi	*Gróbner basis for ideals of the form $I_1(XY)$*
Vijaylaxmi Trivedi	*Hilbert-Kunz density function and Hilbert-Kunz multiplicity*
Vinay Wagh	*LessGenerators – A GAP Package*

Abstracts

Tuesday, 11 October (09:00-10:00)

Speaker : Sudhir Ghorpade, Indian Institute of Technology, Bombay
Title : Hyperplane sections of Schubert varieties over finite fields

Let ℓ, m be integers with $1 \leq \ell < m$ and V be an m-dimensional vector space over a field $\mathbb{F}$. Consider the Grassmannian $G_\ell(V)$ of ℓ-dimensional subspaces of V with its Plücker embedding in $\mathbb{P}(\bigwedge^\ell V)$. Let

$$I(\ell, m) = \{\alpha = (\alpha_1, \ldots, \alpha_\ell) : 1 \leq \alpha_1 < \cdots < \alpha_\ell \leq m\}$$

be the poset with the "Bruhat-Chevalley partial order" given by $\beta \leq \alpha \Leftrightarrow \beta_i \leq \alpha_i \ \forall\ i$. Fix any $\alpha \in I(\ell, m)$ and a partial flag $A_1 \subset \cdots \subset A_\ell$ of subspaces of V such that $\dim A_i = \alpha_i$ for $i = 1, \ldots, \ell$, and let

$$\Omega_\alpha(\ell, m) = \{L \in G_\ell(V) : \dim(L \cap A_i) \geq i \text{ for all } i = 1, \ldots, \ell\}$$

be the corresponding Schubert variety in $G_\ell(V)$. These are projective algebraic varieties defined by equations over $\mathbb{Z}$ and the Grassmannian is a special case with $\alpha_i = m - \ell + i$ for $i = 1, \ldots, \ell$. Now suppose $\mathbb{F}$ is the finite field $\mathbb{F}_q$ with q elements. Using the cellular decomposition, it is easy to see that

$$|\Omega_\alpha(\ell, m)(\mathbb{F}_q)| = n_\alpha := \sum_{\substack{\beta \in I(\ell, m) \\ \beta \leq \alpha}} q^{\delta(\beta)} \quad \text{where} \quad \delta(\beta) := \sum_{i=1}^{\ell} (\beta_i - i).$$

While the embedding, $G_\ell(V) \hookrightarrow \mathbb{P}(\bigwedge^\ell V)$ is nondegenerate (i.e., $G_\ell(V)$ is not contained in any hyperplane of $\mathbb{P}(\bigwedge^\ell V)$), the induced embedding of $\Omega_\alpha(\ell, m)$ in $\mathbb{P}(\bigwedge^\ell V))$ is, in general, not nondegenerate. However, it is not difficult to see (using the Hodge postulation formula) that $\Omega_\alpha(\ell, m)$ embeds nondegenerately in $\mathbb{P}^{k_\alpha - 1}$, where

$$k_\alpha = \det_{1 \leq i, j \leq \ell} \left(\binom{\alpha_j - j + 1}{i - j + 1} \right).$$

We are interested in the maximum number, say e_α, of $\mathbb{F}_q$-rational points that can lie on a section of $\Omega_\alpha(\ell, m)$ by a hyperplane in $\mathbb{P}^{k_\alpha - 1}$. It was conjectured in 1998 that this maximum number is given by $n_\alpha - q^{\delta(\alpha)}$. In the special case of Grassmannians (i.e. when $\alpha_i = m - \ell + i,\ \forall\ i$), this is a classical result of Nogin (1996). The conjecture after being established in several special cases by Chen (2000), Guerra-Vincenti (2004), Ghorpade-Tsfasman (2005), was proved in the affirmative by Xiang (2008). We shall give an alternative proof of the validity of this conjecture. It can be argued that our proof is somewhat simpler and cleaner. Moreover, it paves the way for determining the number of hyperplanes in $\mathbb{P}^{k_\alpha - 1}$ for which the corresponding hyperplane sections of $\Omega_\alpha(\ell, m)$ have the maximum number of $\mathbb{F}_q$-rational points. In this talk, we will provide relevant background and outline these results.

This is a joint work with Prasant Singh.

Tuesday, 11 October (10:30-11:30)

Speaker : Saurav Bhaumik, Indian Institute of Technology, Bombay
Title : Geometric Langlands duality and Hitchin systems

In this survey talk, we will discuss the geometric Langlands duality and its classical limit. The classical limit is formulated in terms of Hitchin systems and anticipates a form of duality between two families of abelian varieties. We will talk about Hitchin systems, spectral covers, cameral covers and abelianisation. Donagi and Pantev settled the question over an open subset of the Hitchin base, while Arinkin and Fedorov formulated and proved a version of the duality for degenerating families of abelian varieties.

Tuesday, 11 October (11:45-12:45)

Speaker : Suhas BN, Indian Institute of Technology, Madras
Title : On the Rationality of Nagaraj-Seshadri Moduli Space

Let X be a reducible nodal curve over an algebraically closed field k of characteristic 0 such that it is a union of two smooth irreducible components X_1 of genus $g_1 \geq 2$ and X_2 of genus $g_2 \geq 2$ meeting exactly at one node p. Let $\mathbf{a} = (a_1, a_2)$ be a tuple of positive rational numbers such that $a_1 + a_2 = 1$; we call this a polarisation on X. Let χ be an integer such that $a_1\chi$ is not an integer. In this setting it is a theorem of Nagaraj-Seshadri [2, Theorem 4.1] that the moduli space $M(2, \mathbf{a}, \chi)$ of semi-stable rank two torsion-free sheaves on X with Euler characteristic χ is a reduced, connected projective scheme with exactly two irreducible components, and when χ is odd, the moduli space is a union of two smooth varieties M_{12} and M_{21} intersecting transversally along a smooth divisor N. Let $\xi = (L_1, L_2)$, where L_1 and L_2 are two invertible sheaves on X_1 and X_2 (of suitable degrees) respectively. Then in [2, Section 7] the analogue of a "fixed determinant moduli space" has been defined and we denote it by $M(2, \mathbf{a}, \chi, \xi)$. It is shown in ([3], [1]) that when χ is odd and $a_1\chi$ is not an integer, $M(2, \mathbf{a}, \chi, \xi)$ is also a reduced, connected projective scheme with exactly two smooth components meeting transversally along a smooth divisor. With this background in mind, we show that each of the irreducible components of $M(2, \mathbf{a}, \chi, \xi)$ is rational.

References

[1] Basu , Suratno; *On a Relative Mumford-Newstead Theorem*, arXiv:1501.07347 [math.AG], To appear in Bulletin des Sciences Mathematiques.

[2] Nagaraj, D.S. ; Seshadri, C.S. ; *Degenerations of moduli spaces of vector bundles on curves -I*, Proc. Indian Acad. Sci (Math. Sci.), Vol-107, No:2, 1997.

[3] Xia, Huashi; *Degenerations of moduli of stable bundles over algebraic curves*, Compositio Math. 98 (1995), no. 3, 305–330.

Tuesday, 11 October (14:30-15:30)

Speaker : Umesh Dubey, Indian Institute of Science, Bangalore
Title : Thick subcategories of singularity category

A classification of thick subcategories arose in many branches of mathematics with wide range of applications e. g. reconstruction theorem of Thomason and Balmer in algebraic geometry. The triangulated category of singularity of a noetherian ring (scheme) is an interesting invariant which captures some information about singularities. In this talk we will briefly explain classification results for the singularity category given by Stevenson, extending the work of Balmer. We will also mention the joint work in progress with Sarang Sane in this direction.

Tuesday, 11 October (16:00-17:00)

Speaker : Prasant Singh, Indian Institute of Technology, Bombay
Title : Maximal linear sections of 2-step flag varieties and quadratic Veronese of Grassmannians

Let V be a vector space of dimension m and $\underline{\ell} = (\ell_1, \ell_2)$ be a pair of integers such that $1 \leq \ell_1 \leq \ell_2 < m$. Denote by $\mathcal{F}(\underline{\ell}; m)$ the collection of all (generalized) partial flags in V of dimension $\underline{\ell}$ i.e.

$$\mathcal{F}(\underline{\ell}; m) = \{(V_1, V_2) \in G_{\ell_1}(V) \times G_{\ell_2}(V) : V_1 \subseteq V_2\}.$$

Plücker map on the projections together with the Segre map of the product gives an embedding of $\mathcal{F}(\underline{\ell}; m)$ into the projective space $\mathbb{P} = \mathbb{P}(\bigwedge^{\ell_1} V \bigotimes \bigwedge^{\ell_2} V)$. It is well known that $\mathcal{F}(\underline{\ell}; m)$ is a projective algebraic subvariety of $\mathbb{P}$ and the dimension of $\mathcal{F}(\underline{\ell}; m)$ is $\delta(\underline{\ell}) = \ell_1(\ell_2 - \ell_1) + \ell_2(m - \ell_1)$. Moreover $\mathcal{F}(\underline{\ell}; m)$ is defined over any finite field $\mathbb{F}_q$ also. The number of $\mathbb{F}_q$-rational points of $\mathcal{F}(\underline{\ell}; m)$ is given by the Gaussian multinomial coefficient $\left[{m \atop \ell_1, \ell_2 - \ell_1, m - \ell_2}\right]_q$.

We consider the following questions:

- Determine the least positive integer $k \leq \binom{m}{\ell_1}\binom{m}{\ell_2}$ such that $\mathcal{F}(\underline{\ell}; m)$ embeds in $\mathbb{P}^{k-1}$.
- Determine the maximal possible number of $\mathbb{F}_q$-rational points in $\mathcal{F}(\underline{\ell}; m) \cap H$, where H varies over hyperplanes in $\mathbb{P}^{k-1}$.

Questions such as these have been considered by several mathematicians in the case of Veronese and Grassmann varieties. For the line-hyperplane incidence variety $\mathcal{F}(\underline{\ell}; m)$ where $\underline{\ell} = (1, m-1)$ these questions were answered by F. Rodier (2003). There has been some partial progress by G. Hana (2005), but the general case appears to be open. We will report some recent progress on both these questions. It may be noted that by permitting $\ell_1 = \ell_2$ we are able to treat the case of 2-step partial flags as well as quadratic Veronese of Grassmannians simultaneously.

This is a joint work with Sudhir R. Ghorpade.

Wednesday, 12 October (09:00-10:00)

Speaker : Rajendra Gurjar, Indian Institute of Technology, Bombay
Title : A simplified proof of a theorem of M. Miyanishi

Let the additive group $G_a := (\mathbf{C}, +)$ act non-trivially and regularly on the polynomial ring in three variables over the complex field. Then the ring of invariants is isomorphic to a polynomial ring in two variables. The original proof of Miyanishi uses a difficult result due to Miyanishi-Tsunoda. We will give a much more accessible proof of this result.

Wednesday, 12 October (10:30-11:30)

Speaker : Krishna Hanumanthu, Chennai Mathematical Institute, Chennai
Title : Positivity of line bundles on blow ups of the complex projective plane

Let X be the blow up of the complex projective plane at finitely many points in the plane. If L is a line bundle on X, then it is interesting to study positivity properties of L such as ampleness, global generation, very ampleness, and more generally, k-very ampleness, where k is a non-negative integer. Apart from their intrinsic interest, these questions have connections to important concepts and conjectures in algebraic geometry. We will discuss these connections and some results when the blown up points are general as well as special.

Wednesday, 12 October (11:45-12:45)

Speaker : Sagar Kolte, Indian Institute of Technology, Bombay
Title : Injectivity of the Vaserstein Symbol

R.A. Rao–W. van der Kallen showed that the Vaserstein symbol $V_{\Gamma(S^3_{\mathbb{R}})}$ from the orbit space of unimodular rows of length three over the coordinate ring of the real three sphere $S^3_{\mathbb{R}}$ modulo elementary action to the elementary symplectic Witt group $W_E(\Gamma(S^3_{\mathbb{R}}))$ was not injective. Dhvanita R. Rao–Neena Gupta gave an uncountable family of singular real threefolds A_α for which the Vaserstein symbol V_{A_α} is not injective. In this paper, we give an uncoutable family of **smooth** real birationally quivalent threefolds A_α for which the Vaserstein symbol V_{A_α} is not injective. This is joint work with Dhvanita R. Rao.

Wednesday, 12 October (14:30-15:30)

Speaker : Dipankar Ghosh, Chennai Mathematical Institute, Chennai
Title : Characterizations of regular local rings via syzygy modules of the residue field

Let R be a commutative Noetherian local ring with residue field k. One of the most influential results in commutative algebra is the result of Auslander, Buchsbaum and Serre: R is regular if and only if projective dimension of k is finite. Note that projective dimension of k is finite if and only if some syzygy module of k is a free module. In this article, we obtain a few new characterizations of regular local rings via syzygy modules of the residue field. We show that if a finite direct sum of syzygy modules of k maps onto 'a semidualizing module' or 'a non-zero maximal Cohen-Macaulay module of finite injective dimension', then R is regular. We also prove that R is regular if and only if some syzygy module of k has a non-zero direct summand of finite injective dimension.

Wednesday, 12 October (16:00-17:00)

Speaker : Gaurab Tripathi, Indian Institute of Technology, Gandhingar
Title : Gróbner basis for ideals of the form $I_1(XY)$

We will costruct Gröbner bases for the ideal $I_1(XY)$, generated by the 1×1 minors of the matrix XY, where X is a matrix whose entries are either indeterminates x_{ij} or 0 and Y is a generic column matrix with entires y_j. The x indeterminates are different from the y indeterminates. We will consider the cases when X is square generic, symmetric and generic $(n+1)\times n$ matrix of indeterminates. This Gröbner basis has been used to compute the Betti numbers and primary decomposition for these ideals, which will be presented by my collaborator Mr Joydip Saha.

Thursday, 13 October (09:00-10:00)

Speaker : Vijaylaxmi Trivedi, Tata Institute of Fundamental Research, Mumbai
Title : Hilbert-Kunz density function and Hilbert-Kunz multiplicity

For a pair (R, I), where R is a standard graded ring R and I is a graded ideal of finite colength, we introduce a new invariant, the *Hilbert-Kunz density function*, which is a limit of a uniformly convergent sequence of real valued compactly supported, piecewise linear and continous functions. We express the Hilbert-Kunz multiplicity, $e_{HK}(R, I)$ as an integral of this function. We show that this function (unlike e_{HK}) satisfies a multiplication formula for the Segre product of rings. As a consequence some known result for e_{HK} of rings hold for e_{HK} of their Segre products. We state (if time permits) a few other applications of this function, like asymptotic behaviour of $e_{HK}(R, I^k)$ as $k \to \infty$, e_{HK} of the Segre product of rings and a possible approach for e_{HK} in characteristic 0.

Thursday, 13 October (10:30-11:30)

Speaker : Amit Hogadi, Indian Institute of Science Education and Research, Pune
Title : Gabber's presentation lemma for finite fields

Gabber's presentation lemma, which plays a fundamental role in A^1 homotopy theory, can be thought of as an algebraic analogue of tubular neighborhood theorem. Unfortunately, there is no published or written proof of this theorem for finite fields. The goal of this talk is to outline a proof of Gabber's presentation lemma for finite fields. This talk is based on joint work with Girish Kulkarni.

Thursday, 13 October (11:45-12:45)

Speaker : Rajiv Kumar, Indian Institute of Technology, Bombay
Title : Herzog-Kuhl Equations and its Applications

In 1984, Herzog and K̈hl give relations of Betti numbers and their shifts for pure Cohen-Macaulay modules over polynomial rings over a field k. We generalize their result for pure modules of finite projective dimension over standard graded k-algebras. We give the following applications for the Herzog-K̈hl equations; we characterize Cohen-Macaulay rings in terms of modules with pure resolutions. Secondly, we find a class of modules which satisfies the Buchsbaum-Eisenbud-Horrocks conjecture.

Thursday, 13 October (14:30-15:30)

Speaker : Rakesh Reddy, Indian Institute of Science Education and Research, Thiruvananthapuram

Title : Relative Hilbert Coefficients

Let $(A, \mathfrak{m})$ be a Cohen-Macaulay local ring of dimension d and let $I \subseteq J$ be two $\mathfrak{m}$-primary ideals with I a reduction of J. For $i = 0, \ldots, d$ let $e_i^J(A)$ ($e_i^I(A)$) be the i^{th} Hilbert coefficient of J (I) respectively. We call the number $c_i(I, J) = e_i^J(A) - e_i^I(A)$ the i^{th} relative Hilbert coefficient of J with respect to I. If $G_I(A)$ is Cohen-Macaulay then $c_i(I, J)$ satisfy various constraints. We also show that vanishing of some $c_i(I, J)$ has strong implications on $depth G_{J^n}(A)$ for $n \gg 0$.

Thursday, 13 October (16:00-17:00)

Speaker : Joydip Saha, Ramakrishna Mission Vivekananda University, Belur

Title : Primary decomposition and Betti numbers of ideals of the form $I_1(XY)$

We consider ideals of the form $I_1(XY)$, generated by the 1×1 minors of the matrix XY, where X is a matrix whose entries are either indeterminates x_{ij} or 0 and Y is a generic column matrix with entries y_j. The x indeterminates are different from the y indeterminates. We know Gröbner bases for $I_1(XY)$ for the cases when X is square generic, symmetric and generic $(n + 1) \times n$ matrix of indeterminates. We will use this Gröbner bases for $I_1(XY)$ to compute primary decompositions and Betti numbers for these ideals.

Friday, 14 October (09:00-10:00)

Speaker : Tony Joseph Puthenpurakal, Indian Institute of Technology, Bombay

Title : Symmetries and connected components of the AR-quiver

Let $(A, \mathfrak{m})$ be a commutative complete equicharacteristic Gorenstein isolated singularity of dimension d with $k = A/\mathfrak{m}$ algebraically closed. Let $\Gamma(A)$ be the AR (Auslander-Reiten) quiver of A. Let $\mathcal{P}$ be a property of maximal Cohen-Macaulay A-modules. We show that some naturally defined properties $\mathcal{P}$ define a union of connected components of $\Gamma(A)$. So in this case if there is a maximal Cohen-Macaulay module satisfying $\mathcal{P}$ and if A is not of finite representation type then there exists a family $\{M_n\}_{n\geq 1}$ of maximal Cohen-Macaulay indecomposable modules satisfying $\mathcal{P}$ with multiplicity $e(M_n) > n$. Let $\underline{\Gamma}(A)$ be the stable quiver. We show that there are many symmetries in $\underline{\Gamma}(A)$. Furthermore $\underline{\Gamma}(A)$ is isomorphic to its reverse graph. As an application we show that if $(A, \mathfrak{m})$ is a two dimensional Gorenstein isolated singularity with multiplicity $e(A) \geq 3$ then for all $n \geq 1$ there exists an indecomposable self-dual maximal Cohen-Macaulay A-module of rank n.

Friday, 14 October (10:30-11:30)

Speaker : Yashonidhi Pandey, Indian Institute of Science Education and Research, Mohali

Title : Brauer group of the moduli stack and space of parahoric torsors

We will discuss the general proof strategy of computation of the Brauer group.

Friday, 14 October (11:45-12:45)

Speaker : Parangama Sarkar, Indian Institute of Technology, Bombay

Title : Local cohomology of multi-Rees algebras, joint reduction numbers and product of complete ideals

We find conditions on the local cohomology modules of multi-Rees algebras of admissible filtrations which enable us to predict joint reduction numbers. As a consequence we are able to prove a generalisation of a result of Reid-Roberts-Vitulli in the setting of analytically unramified local rings for completeness of power products of complete ideals.

Friday, 14 October (14:30-15:30)

Speaker : Shiv Datt Kumar, Motilal Nehru National Institute of Technology, Allahabad

Title : Depth and Dimension in Symbolic Rees Algebra of Modules

This is joint work with Priti Singh. The symbolic Rees algebra was introduced in connection with Zariski's generalization of Hilbert's fourteenth problem. Rees gave a counter example to Zariski's conjecture by constructing a non-Noetherian symbolic Rees algebra. In this paper, we study the symbolic Rees algebra of modules within a fairly general frame work and give necessary conditions for the symbolic Rees algebra of modules to be Noetherian. We describe some notions for finding depth and dimension of components of Rees algebra of modules. Following are some results:

Lemma 1 *Let R be a Noetherian ring and $E \subset G \simeq R^e$, $e > 0$ be an R-submodule of G.*

1. *Then $Min(G^n/E^n) = Min(G/E)$ for any $n \geq 1$.*
2. *If the symbolic Rees algebra $R_s(E)$ is a Noetherian ring, then there exist $k > 0$ such that $E^{(k)n} = E^{(kn)}$ for all $n \geq 1$. Converse holds if R is a Nagata ring.*

Proposition 2 *Let R be a Noetherian ring with $dim(R) = d > 0$ and $E \subset G \simeq R^e$, $e > 0$ be an R-module with $dim(G/E) > 0$. Then*

1. *$depth\left(\dfrac{G^n}{E^{(n)}}\right) \geq 1$ for any $n \geq 1$.*

2. *If R is a Cohen-Macaulay, then $1 \leq depth(E^{(n)}) \leq depth(G^n/E^{(n)}) + 1 \leq d - 1$.*

Theorem 3 *Let $(R, \mathfrak{m})$ be a Noetherian local ring with $dim(R) = d > 0$ and $G \simeq R^e$ be a free R-module with rank $e > 0$. Assume that $E \subset G$ is an R-submodule and symbolic Rees algebra $R_s(E)$ is a Noetherian ring. Then there exists $k > 0$ such that $l(E^{(k)}) \leq d + e - 2$.*

We also give an example to show that $l(E^{(k)}) \leq d + e - 2$ for some $k > 0$ but we do not always have a reduction generated by $d + e - 2$ elements, even when $R_s(E)$ is a Noetherian ring and $R/\mathfrak{m}$ is an infinite residue field.

Proposition 4 *Let $E \subset G \simeq R^e$, $e > 0$ be an ideal module, $R_s(E)$ is a Noetherian ring and $depth(G^n/E^n) = d - ht(\mathbb{F}_e(E))$ for infinitely many n. Then $E^{(k)}$ is a equimultiple module for some $k > 0$.*

Friday, 14 October (16:00-16:30)

Speaker : Selvaraja S, Indian Institute of Technology, Madras
Title : Regularity of powers of bipartite graphs

Let G be a finite simple graph and $I(G)$ denote the corresponding edge ideal. For all $s \geq 1$, we obtain upper bounds for $\operatorname{reg}(I(G)^s)$ for bipartite graphs. We then compare the properties of G and G', where G' is the graph associated with the polarization of the ideal $(I(G)^{s+1} : e_1 \cdots e_s)$, where $e_1, \ldots e_s$ are edges of G. Using these results, we explicitly compute $\operatorname{reg}(I(G)^s)$ for several subclasses of bipartite graphs.

Friday, 14 October (16:30-17:00)

Speaker : Arindam Dey, Indian Institute of Technology, Guwahati
Title : On the module of derivations of certain rings of invariants

Let $\mathbf{k}$ be an algebraically closed field of characteristic 0. Let G be a finite cyclic subgroup of $GL_m(\mathbf{k})$ having no non-trivial pseudo-reflections. Let R be the ring of invariants obtained by the linear action of G on $\mathbf{k}[X_1, \ldots, X_m]$. We give an algorithm to find a generating set for $Der\ R$. We also give an upper bound for $\mu(Der\ R)$.

Saturday, 15 October (09:00-10:00)

Speaker : Jai Laxmi, Indian Institute of Technology, Bombay
Title : Decomposing Gorenstein Rings as Connected Sums

In 2012, Ananthnarayan, Avramov and Moore give a new construction of Gorenstein rings from two Gorenstein local rings, called their connected sum. Given a Gorenstein Artin ring, we investigate conditions on the ring which force it to be indecomposable as a connected sum. We will see characterizations for Gorenstein Artin local rings to be decomposable. Finally, we will show that the indecomposable components appearing in the connected sum decomposition are unique up to isomorphism.

Saturday, 15 October (10:30-11:30)

Speaker : Husney Parvez Sarwar, Tata Institute of Fundamental Research, Mumbai
Title : K-theory of monoid algebras

This is a joint work with Amalendu Krishna. We answer a question of Gubeladze about K_1 of a affine singular scheme (monoid algebra) in affirmative. At the end, we give an application of our result to a question of Lindel.

Saturday, 15 October (11:45-12:45)

Speaker : Ananthnarayan Hariharan, Indian Institute of Technology, Bombay
Title : Associated Graded Rings and Connected Sums

In 2012, Ananthnarayan, Avramov and Moore give a new construction of Gorenstein rings from two Gorenstein local rings, called their connected sum. In this talk, we will see conditions on the associated graded ring of a Gorenstein Artin ring which force it to be a connected sum. As a consequence, we obtain results about its Poincare series and minimal number of generators of its defining ideal. This is joint work with E. Celikbas, Jai Laxmi, and Z. Yang.

Saturday, 15 October (14:30-15:30)

Speaker : Jyoti Singh, Visvesvaraya National Institute of Technology, Nagpur
Title : Generalized Eulerian property of graded local cohomology modules

Let $R = K[X_1, \ldots, X_n]$, where K is a field of characteristic zero and R is standard graded. Let $\mathfrak{m} = (X_1, \ldots, X_n)$ and let E be the *injective hull of $R/\mathfrak{m}$. Let $A_n(K)$ be the n^{th} Weyl algebra over K. If $\mathcal{T}$ is graded Lyubeznik functor on $^*Mod(R)$, then we show that $\mathcal{T}(R)$ is generalized Eulerian $A_n(K)$-module. As an application, we show that $H^i_{\mathfrak{m}}\mathcal{T}(R) \cong E(n)^{a_i}$ for some $a_i \geq 0$. (This is a characteristic zero version of a result due to Ma and Zhang in characteristic $p > 0$).

Saturday, 15 October (16:00-17:00)

Speaker : Vinay Wagh, Indian Institute of Technology, Guwahati
Title : LessGenerators – A GAP Package

A GAP package called "LessGenerators" has been developed in collaboration with Mohamed Barakat. The package has a complete implementation of the Quillen-Suslin algorithm, for polynomial rings over a "computable" field of characteristic zero. The aim of the package LessGenerators is to provide a tool for finding a minimal generating set for a given module. The implementation in this package is mostly based on the algorithmic proof of Quillen-Suslin theorem by Logar and Sturmfels.

The package "LessGenerators" is part of the homalg project. The structure of the package is in sync with the base concept of the homalg project and provides universal implementation in the sense of CASs. i.e. it can use any computer algebra systems supported by the homalg project for ring arithmetic and lower level computations. The homalg package "LocalizeRingForHomalg" supports local rings obtained via localization of the baserings at maximal ideals. However Logar-Sturmfels algorithm requires localization at prime ideals. This has been achieved by providing partial support to the rings of the type $k[X_1, \ldots, X_{n-1}]_{\wp}[X_n]$.

For the output, given a unimodular row (over a polynomial ring over a computable field), the function produces an invertible matrix whose first row is same as the input (in other words, its completion to an invertible matrix), as well as the matrix of transformation. Several heuristics (short-cuts) have been implemented, mostly based on other theoretical results. These heuristics help in speeding up the computations.

Generalization of Key Lemma on Suslin Matrices

Selby Jose

Abstract: In the theory of Suslin matrices, by analyzing the action of elementary matrices on unimodular rows, we obtain the Key Lemma

$$\begin{aligned} S_r(vE_{i1}(-\lambda), wE_{1i}(\lambda)) &= S_r(e_1, e_1E_{1i}(\lambda))^{top} S_r(v,w) S_r(e_1, e_1E_{1i}(\lambda))^{bot} \\ S_r(vE_{1i}(\lambda), wE_{i1}(-\lambda)) &= S_r(e_1E_{1i}(\lambda), e_1)^{bot} S_r(v,w) S_r(e_1E_{1i}(\lambda), e_1)^{top} \end{aligned}$$

We generalize this key lemma by analysing the action of $\mathrm{SL}_{r+1}(R)$ on unimodular rows of lenght $r+1$ and prove that

$$S_r(v\sigma, w{\sigma^t}^{-1}) = AS_r(v,w)B$$

for some $A, B \in SL_{2^r}(R)$.

Generalization of Key Lemma on Suslin Matrices

Selby Jose

Abstract: In the theory of Suslin matrices, by analyzing the action of elementary matrices on unimodular rows, we obtain the Key Lemma

$$\begin{aligned} S_r(vE_{i1}(-\lambda), wE_{1i}(\lambda)) &= S_r(e_1, e_1E_{1i}(\lambda))^{top} S_r(v,w) S_r(e_1, e_1E_{1i}(\lambda))^{bot} \\ S_r(vE_{1i}(\lambda), wE_{i1}(-\lambda)) &= S_r(e_1E_{1i}(\lambda), e_1)^{bot} S_r(v,w) S_r(e_1E_{1i}(\lambda), e_1)^{top} \end{aligned}$$

We generalize this key lemma by analysing the action of $\mathrm{SL}_{r+1}(R)$ on unimodular rows of lenght $r+1$ and prove that

$$S_r(v\sigma, w{\sigma^t}^{-1}) = AS_r(v,w)B$$

for some $A, B \in SL_{2^r}(R)$.