TITLES AND ABSTRACTS

1. December 5th, 2017

R V Gurjar (IIT Bombay)

Time: 09:30–10:30

Title: *On the Fundamental Group of a Smooth Projective Surface with a Finite Group of Automorphisms*

Abstract: In this article we prove some results on fundamental groups for some classes of fibered smooth projective algebraic surfaces with a finite group of automorphisms. The methods actually compute the fundamental groups of the surfaces under study upto finite index.

The corollaries include an affirmative answer to Shafarevich conjecture on holomorphic convexity, Nori's well-known question on fundamental groups and free abelianness of second homotopy groups for these surfaces.

We also prove a theorem that bounds the multiplicity of the multiple fibers of a fibration for any algebraic surface with a finite group of automorphisms G in terms of the multiplicities of the induced fibration on X/G.

If X/G is a $\mathbf{P}^1$-fibration, we show that the multiplicty actually divides $|G|$. This theorem on multiplicity, which is of independent interest, plays an important role in our proofs.

This is a joint work with B.P. Purnaprajna.

Sudhir Ghorpade (IIT Bombay)

Time: 10:35–11:35

Title: *Hypersurfaces in weighted projective spaces over finite fields*

Abstract: The weighted projective spaces are fascinating objects. On the one hand, they are very analogous to classical projective spaces, but on the other hand, they are often difficult to deal with, partly since they can admit singularities. We consider the question of determining the maximum number of points that can lie on a hypersurface of a given degree in a weighted projective space over a given finite field. in other words, we ask for the maximum number of zeros that a weighted homogeneous polynomial of a given degree can have in the corresponding weighted projective space over the given finite field. In the case of classical projective spaces, this question has been answered by J.-P. Serre. In the case of weighted projective spaces, we give some conjectures and partial results.

This is a joint work with Y. Aubry, W. Castryck, G. Lachaud, M. E. O'Sullivan and S. Ram.

Arindam Banerjee (RKM Vivekananda University)

Time: 11:50–12:20

Title: *Homological Algebra of the Path Ideals of Finite Simple Graphs*

Abstract: Path ideals (and their special instance edge ideals) are square free monomial ideals associated to finite simple graphs. The combinatorics of those graphs appear to have interesting effects on the homological algebra of these ideals. In this talk we shall discuss some of the known results, some recent developments and some open problems that are expected to be studied in near future.

A V Jayanthan (IIT Madras)

Time: 12:25–12:55

Title: *On the Vasconcelos inequality for the fiber multiplicity of modules.*

Abstract: Let $(R,\mathfrak{m})$ be a Noetherian local ring of dimension $d > 0$ with infinite residue field. Let M be a finitely generated proper R-submodule of a free R-module F with $\ell(F/M) < \infty$ and having rank r. In this article, we study the fiber multiplicity $f_0(M)$ of the module M. We prove that if $(R,\mathfrak{m})$ is a two

dimensional Cohen-Macaulay local ring, then $f_0(M) \leq br_1(M) - br_0(M) + \ell(F/M) + \mu(M) - r$, where $br_i(M)$ denotes the ith Buchsbaum-Rim coefficient of M. This is a joint work with R. Balakrishnan.

Chetan Balwe (IISER Mohali)

Time: 14:00–15:00
Title: $\mathbb{A}^1$-*connected components of reductive groups*
Abstract: This talk will present some results regarding the sheaf of $\mathbb{A}^1$-connected components of reductive groups. This is done by studying $\mathbb{A}^1$-homotopies on the groups and exploring the relationship between the sheaf of A1-chain connected components. As a result, we show that, over the base field, the $\mathbb{A}^1$-connected compoents of a semisimple, simply connected algebraic group are the same as its R-equivalence classes. We will also characterize $\mathbb{A}^1$-connected reductive groups over a field of characteristic zero. This talk is based on joint work with Anand Sawant.

Sarang Sane (IIT Madras)

Time: 15:05–15:35
Title: *A criterion for local rings to be Cohen-Macaulay based on a derived equivalence.*
Abstract: Let R be a commutative, noetherian ring. We will mention a trick using Koszul complexes leading to an equivalence of certain derived categories of R-modules. This will allow us to classify Cohen-Macaulay rings in terms of derived equivalences. This is joint work with William Sanders.

Krishna Hanumanthu (CMI)

Time: 15:50–16:20
Title: *Seshadri Constants*
Abstract: Seshadri constants are invariants of line bundles on a projective variety. They were defined in 1990s by Demailly who was inspired by an ampleness criterion of Seshadri. They have given rise to very interesting questions and a lot of current research is focused on these questions. Part of this talk will be expository and will aim to give an overview of the landscape of current research on Seshadri constants. We will also talk about some recent results in the case of surfaces.

A A Ambily (Cochin University of Science and Technology)

Time: 16:25–16:55
Title: *Normality of DSER elementary orthogonal group*
Abstract: This talk is based on a joint work with Ravi A. Rao.

Let (Q, q) be an inner product space over a commutative ring R in which 2 is invertible. A. Roy introduced the elementary orthogonal group $\mathrm{EO}_R(Q \perp H(R)^m)$ of a quadratic space with a hyperbolic summand over a commutative ring R. This construction of Roy generalized the earlier work of Dickson-Siegel-Eichler-Dieudonne over fields.

Consider the DicksonSiegelEichlerRoys subgroup of the orthogonal group $\mathrm{O}_R(Q \perp H(R)^m)$, with $\mathrm{rank}\, Q = n \geq 1$ and $m \geq 1$. In this talk, we shall discuss the normality of $\mathrm{EO}_R(Q \perp H(R)^m)$ in $\mathrm{O}_R(Q \perp H(R)^m)$, for all $m \neq 2$. In particular, when $\mathrm{rank}\, Q = r$ for $r \geq 1$ and $m \geq 2$, the group $\mathrm{EO}_R(Q \perp H(R)^m) = \mathrm{EO}_{2(r+m)}(R)$ is normal in $\mathrm{O}_R(H(R)^{r+m})$. We also prove that the DSER group $\mathrm{EO}_R(Q, H(P))$ is a normal subgroup of $\mathrm{O}_R(Q \perp H(P))$, where Q and $H(P)$ are quadratic spaces over a commutative ring R, with $\mathrm{rank}(Q) \geq 1$ and $\mathrm{rank}(P) \geq 2$.

Selvaraja S (IIT Madras)

Time: 17:00–17:30
Title: *Regularity of powers of edge ideals*
Abstract: Behaviour of the Castelnuovo-Mumford regularity of powers of homogeneous ideals of polynomial rings have been an active area of research in the past decade. In this talk, first I will discuss about the regularity of powers of edge ideals of some classes of finite simple graphs. At the end, my recent work with Jayanthan about regularity of powers of edge ideals of very well-covered (unmixed) graphs will be presented.

2. December 6th, 2017

Neena Gupta (ISI Kolkata)

Time: 09:30–10:30

Title: *On Characterizations of Affine spaces and Cancellation Problems.*

Abstract: Let k be an algebraically closed field of characteristic zero. In this talk we shall discuss various known characterizations of the affine spaces $\mathbb{A}_k^1$, $\mathbb{A}_k^2$ and $\mathbb{A}_k^3$ and their significance in the context of the cancellation problems on affine spaces.

We shall also mention new algebraic characterizations of the affine 2-space and the affine 3-space over k in terms of a variant of the Makar-Limanov invariant. These recent results have been obtained in a joint work with Nikhilesh Dasgupta.

A J Parameswaran (TIFR)

Time: 10:35–11:35

Title: *Automorphism Group of a Bott-Samelson-Demazure-Hansen (BSDH) Variety*

Abstract: Let G be a simple, adjoint, algebraic group over the field of complex numbers, B be a Borel subgroup of G containing a maximal torus T of G, w be an element of the Weyl group W and $X(w)$ be the Schubert variety in G/B corresponding to w. Let $Z(w, \underline{i})$ be the Bott-Samelson-Demazure-Hansen variety corresponding to a reduced expression $\underline{i}$ of w. Here we compute the connected component $Aut^0(Z(w, \underline{i}))$ of the automorphism group of $Z(w, \underline{i})$ containing the identity automorphism.

This is a joint work with S. Senthamarai Kannan and B. Narasimha Chary.

Ananthnarayanan H (IIT Bombay)

Time: 11:50–12:20

Title: *Boij-Soderberg theory over standard graded rings*

Abstract: We begin with the motivation behind the Boij-Soderberg conjectures (2008), followed by a quick word on the techniques used in their resolution by Eisenbud-Schreyer (2009). In the rest of the talk, we will see what works more generally, and try to identify the difficulties in applying the same techniques in other cases. Finally, we will see a case where the same strategy works, which is joint work with Rajiv Kumar.

Krishna Kaipa (IISER Pune)

Time: 12:25–12:55

Title: *The questions of Segre in finite geometry.*

Abstract: In 1955 Beniamino Segre posed three foundational questions in finite geometry which are still open, although partial answers are known. These questions were posed soon after Segre obtained his famous result that a set of $q + 1$ points in general position in the projective plane $PG(2, q)$ with q odd, lie on an irreducible conic. In this talk we will discuss two of these questions. To state Segre's questions, let us define an n-arc in $PG(m, q)$ to be a set of n-points in m-dimensional projective space, such that no $m + 1$ of these points lie on a hyperplane.

Q1: What is the maximum size of an arc in $PG(m, q)$?

Q2: For which m,q is every $q + 1$-arc of $PG(m, q)$,the point set of a rational normal curve?

We will also present a new proof of the best general result about these questions, using the combinatorial nullstellensatz of Noga Alon.

Utsav Choudhury (IISER Kolkata)

Time: 14:00–15:00

Title: *Some contractibility and rigidity results in unstable motivic homotopy theory*

Abstract: I will present some results related to the rigidity of connected component sheaf in unstable motivic homotopy theory. Using obstruction theory I will describe a relevant rigidity property of truncation functors arising from Postnikov's tower in unstable motivic homotopy theory.We will see that presence of non-constant units on a affine smooth algebra is a obstruuction to connectivity and hence contractibility.

Viji Thomas (IISER Thiruvananthapuram)

Time: 15:05–15:35
Title: *The Bazzoni-Glaz Conjecture*

Abstract: The Bazzoni-Glaz conjecture is a conjecture related to a conjecture of Kaplansky on Gaussian rings. In this talk we will outline a proof of the Bazzoni-Glaz Conjecture on the Weak Global dimension of Gaussian rings.

Sampat Sharma (TIFR)

Time: 15:50–16:20
Title: *Homotopy and commutativity principle*

Abstract: I will give an evidence for the principle that a (special linear, symplectic, transvection) matrix over a commutative ring which is homotopic to the identity will commute up to an elementary matrix with all elements of the (respective) group. We can generalize this principle that a (special linear, symplectic) matrix over a commutative ring which is homotopic to identity will commute up to an elementary matrix with all $(n \times m)$ right invertible matrices. Then we will deduce one of Vaserstein's lemaa which states that if $\delta \in SL_n(A)$ and $V \in Um_{n,m}(A)$. Then $\delta V = V\sigma$ for some $\sigma \in SL_m(A)$ such that $(\delta^{-1} \perp \sigma) \in E_{n+m}(A)$.

Neeraj Kumar (IIT Bombay)

Time: 16:25–16:55
Title: *Syzygies of powers of a certain squarefree monomial ideals*

Abstract: In this talk, I will present an overview of our recent work on the graded minimal free resolutions of powers of special classes of squarefree monomial ideals. In this work, we describe explicitely all the syzygy relations of graded minimal free resolution. As an application, we derive expressions for Hilbert series, Betti numbers and the regularity of powers of cover ideals of some complete multipartite graphs. This talk is based on a joint work with A.V. Jayanthan

Kriti Goel (IIT Bombay)

Time: 17:00–17:30
Title: *Tight Hilbert Polynomials*

Abstract: We introduce the tight Hilbert polynomial of an m-primary ideal in a Noetherian local ring of prime characteristic $p > 0$. The tight Hilbert polynomial characterises F-rational local rings. We establish a characteristic p version of Huneke-Itoh intersection theorem and use it to find the tight Hilbert polynomial of ideals with tight reduction number at most 2.
(Joint work with Vivek Mukundan and Prof. J. K. Verma.)

3. December 7th, 2017

Madhusudan M (IIT Bombay)

Time: 09:30–10:30
Title: *Frobenius Numbers: A Commutative Algebraic Perspective*

Abstract: For a natural number k, the k-th (generalised) Frobenius number of relatively prime natural numbers $(a_1, \ldots, a_n)$ is the largest natural number that cannot be written as a non-negative integral combination of $(a_1, \ldots, a_n)$ in k distinct ways. We study the k-th Frobenius number from a commutative algebraic perspective. We interpret the k-th Frobenius number in terms of the Castelnuovo-Mumford regularity of certain modules associated to $(a_1, \ldots, a_n)$. We study these modules in detail and using this study, show that the sequence of generalised Frobenius numbers form a finite difference progression i.e., a sequence whose set of successive differences form a finite set. This talk is based on a joint work with Ben Smith.

Manoj Kummini (CMI)

Time: 10:35–11:35
Title: *F-rationality of Rees Algebras*
Abstract: Let R be a three-dimenensional finite-type domain over a field of prime characteristic, and I an ideal of R primary to a maximal ideal. Assume that R has rational singularities. We will show that if the blow-up of $\operatorname{Spec}(R)$ along I is F-rational, then for all sufficiently large integer N, the Rees algebra $R[I^N]$ is F-rational. This is joint work with M. Koley.

Anjan Gupta (Universita degli Studi di Genova)

Time: 11:50–12:20
Title: *Poincare Series of Modules over Local Rings*
Abstract: Let R be a local ring with maximal ideal $\mathfrak{m}$ and residue field k. The Poincaré series of a finitely generated module M over R is defined as the formal power series

$$P_M^R(t) = \sum_{i \geq 0} \dim_k \operatorname{Tor}_i^R(M,k)t^i \in \mathbb{Z}[|t|].$$

In the 1950s Serre and Kaplansky asked whether the Poincaré series $P_k^R(t)$ is always a rational function. An example of an irrational Poincaré series was first constructed by Anick. Nevertheless there are large classes of rings over which all finitely generated modules have rational Poincaré series.

Now assume that R is Artinian. The ring R is called stretched if $\mathfrak{m}^2$ is principal and almost stretched if $\mathfrak{m}^2$ is minimally generated by two elements. In this talk, I will present new examples of local rings R such that all finitely generated modules over R have rational Poincaré series sharing a common denominator. The examples include (I) stretched Artinian rings and (II) almost stretched Gorenstein Artinian rings.

Auslander-Reiten conjecture states that if $\operatorname{Ext}_R^{>0}(M, M \oplus R) = 0$ for a finitely generated module M over a local ring R, then M is free. Using the rationality of Poincaré series and a result of Şega, I will prove that Auslander-Reiten conjecture has an affirmative answer for rings of type (I) and (II).

1. *Detecting Koszulness and related homological properties from the algebra structure of Koszul homology*; Amanda Croll, Roger Dellaca, Anjan Gupta, Justin Hoffmeier, Vivek Mukundan, Denise Rangel Tracy, Liana M. Şega, Gabriel Sosa, Peder Thompson.
2. *A connection between the good property of an Artinian Gorenstein local ring and that of its quotient modulo socle,*
   Anjan Gupta.

Mitra Koley (CMI)

Time: 12:25–12:55
Title: *F-rationality of Rees and Extended Rees algebras.*
Abstract: *F*-rationality of Rees algebras was studied by Hara, Watanabe and Yoshida. They showed if Rees algebra is *F*-rational, then so is extended Rees algebra. Converse of this was given as a conjecture. We will see the converse. As a corollary we get their another result that Rees algebras of integrally closed ideals in two dimensional excellent *F*-rational local rings are *F*-rational. This is a joint work with Manoj Kummini.

Mrinal Das (ISI Kolkata)

Time: 14:00–15:00
Title: *Stably free modules over smooth real affine algebras.*
Abstract: Let R be a smooth affine algebra of dimension d over the field real numbers. We shall talk about the structure of isomorphism classes of stably free R-modules of rank d.

Md Ali Zinna (NISER)

Time: 15:05–15:35
Title: *A monic inversion principle*
Abstract: I this talk, we will discuss about the following interesting question, which is, if fact, an analogue of the affine Horrocks' theorem.

Let R be a commutative Noetherian ring of dimension $d \geq 2$ and $I \subset R[T]$ be an ideal of height n such that $\mu(I/I^2) = n$. Let $I = (f_1, \cdots, f_n) + I^2$ be given. Assume that $IR(T) = (g_1, \cdots, g_n)$ such that $f_i - g_i \in IR(T)^2$, where the ring $R(T)$ is obtained from $R[T]$ by inverting all the monic polynomials. Then, do there exist $h_1, \cdots, h_n \in I$ such that $I = (h_1, \cdots, h_n)$ with $f_i - h_i \in I^2$?

Nikhilesh Dasgupta (ISI Kolkata)

Time: 15:50–16:20
Title: *Nice Derivations over Principal Ideal Domains*
Abstract: Let k be a field of characteristic zero, R a k-domain, $B := R^{[n]}$ and m a positive integer $\leq n$. In this talk, I will consider locally nilpotent derivations D on B, which satisfy $D^2(T_i) = 0$ for all $i \in \{1, \ldots, m\} \subset \{1, \ldots, n\}$ for some coordinate system $(T_1, T_2, \ldots, T_n)$ of B. For convenience, we shall call such a derivation D a *quasi-nice derivation*. In the case $m = n$, such a D is called a *nice derivation*. The case when $B = k^{[3]}$ was investigated by Z. Wang. He showed that rank D is less than 3 for the cases $(m, n) = (2, 3), (3, 3)$ and that rank $D = 1$ when D is a nice derivation (i.e., for $(m, n) = (3, 3)$). Moreover, Daigle proved that the rank of D is less than 3 even in the case $(m, n) = (1, 3)$.

In this talk, we shall see how far one can extend the results of Wang and Daigle to $R[X, Y, Z]$, where R is a Noetherian domain containing $\mathbb{Q}$.

Shreedevi Masuti (University of Genova)

Time: 16:25–16:55
Title: *Hilbert Functions of Level Algebras*
Abstract: In this talk, we will discuss possible Hilbert functions of Artinian level K-algebras (need not be graded) of codimension 3. We give a complete characterization of numerical sequences that can occur as the Hilbert functions of level (in particular, Gorenstein) rings in some cases. Moreover, we construct an Artinian Gorenstein K-algebra which is not analytically isomorphic to its associated graded ring (in particular, is not graded) in certain cases. This is my joint work with M. E. Rossi.

Saloni Singh (CMI)

Time: 17:00–17:30
Title: *Bounding Hilbert coefficients of parameter ideals*
Abstract: Let $(R, \mathfrak{m})$ be a Noetherian local ring of dimension $d > 0$ and Q a parameter ideal in R. When R is a generalized Cohen-Macaulay ring, there are well known bounds for the first Hilbert coefficient $e_1(Q)$ in terms of local cohomology modules $H^i_{\mathfrak{m}}(R)$. It is also known that these bounds are sharp. Concerning the higher coefficients, Goto and Ozeki proved that the sets $\{e_i(Q) : Q \text{ is parameter ideal in } R\}_{1 \leq i \leq d}$ have uniform bounds if and only if R is generalized Cohen-Macaulay. However, their bounds for $e_i(Q)$ are very huge except when $i = 2$. So far, no sharp bounds are known for $e_3(Q)$ for parameter ideals Q. We prove that $e_3(Q)$ is non-positive when R has depth at least $d - 1$. With some additional hypothesis on the depth of associated graded ring $G(Q)$, we provide uniform lower and upper bounds for $e_i(Q)$ for $2 \leq i \leq d$. This is a joint work with Anupam Saikia.

4. December 8th, 2017

Vijaylaxmi Trivedi (TIFR)

Time: 09:30–10:30
Title: *Towards Hilbert-Kunz density functions and multiplicities in Characteristic 0*
Abstract: For a pair (R, I), where R is a standard graded ring of dimension d over an algebraically closed field of characteristic 0 and I is a graded ideal of finite colength, we prove that the existence of

$$\lim_{p \to \infty} e_{HK}(R_p, I_p) := \lim_{p \to \infty} \lim_{m \to \infty} \ell(R_p/I_p^{[p^m]})/p^{md}$$

is equivalent, for any fixed $m \geq d - 1$, to the existence of

$$\lim_{p \to \infty} \ell(R_p/I_p^{[p^m]})/p^{md}.$$

Moreover, if such a limit exists then both the limits are equal.

For $d = 2$, this result was proved by H. Brenner-Jinja Li-C. Miller.

To prove the result, in higher dimensions, we use the theory HK density functions and first prove the analogous statement for HK density functions.

This allows us to prove the existence of $e_{HK}^{\infty}(R, I)$ in many new cases, e.g., when $\operatorname{Proj} R$ is a Segre product of curves.

Usha Bhosle (IISc)

Time: 10:35–11:35
Title: *Vector bundles on rational ruled surfaces.*

Abstract: Vector bundles on nonsingular ruled surfaces have been studied by many authors. We study moduli spaces of slope-semistable vector bundles of fixed rank and fixed Chern classes on a singular rational ruled surface. We show that under certain conditions, these moduli spaces are irreducible, smooth and rational (when nonempty). We also prove that they are nonempty in some cases. We show that for a rational ruled surface defined over real numbers, the above moduli space is rational as a variety defined over real numbers. This is joint work with Indranil Biswas.

Indranath Sengupta (IIT Gandhinagar)

Time: 11:50–12:20
Title: *A Gröbner basis method to compute Primary decomposition of certain determinantal ideals*

Abstract: In a recent work with Joydip Saha and Gaurab Tripathi we have computed the primary decomposition of certain determinantal ideals using the notion of *complete irreducibility* and Gröbner basis. We will be discussing the main results and show how Gröbner can be used effectively for this class of ideals. Some open problems will be presented at the end.

Subham Sarkar (TIFR)

Time: 12:25–12:55
Title: *Iterated integrals and Special values of Hypergeometric functions*

Abstract: In this talk we discuss certain extensions of mixed Hodge structures which are coming from the mixed Hodge structure on the graded quotients of the group ring of the Fundamental group of a smooth projective pointed curve. Such extensions are computed using iterated integrals. In a very special case those iterated integral evaluated as a special values of hypergeometric functions.

V Srinivas (TIFR)

Time: 14:00–15:00
Title: *Frobenius action on local cohomology and the Hodge filtration*

Abstract: In this talk, based on joint work with S. Takagi, I will discuss a variation of the "weak ordinarity conjecture" (which is in fact a consequence of it). For a d dimensional isolated CM singularity in char. 0, this new conjecture relates the nilpotency of the Frobenius action on local cohomology in degrees $< d$ of the mod p reductions, and the vanishing of the 0-th graded piece of the Hodge filtration on the vanishing cycle cohomology, in char. 0. I will also discuss some known cases.

Mandira Mondal (TIFR)

Time: 15:05–15:35
Title: *Hilbert-Kunz density function and asymptotic Hilbert-Kunz multiplicity on projective toric varieties*

Abstract: For a pair (R, I) where R is a Noetherian standard graded ring over an algebraically closed field of characteristic $p > 0$, and I is a homogeneous ideal of finite colength, Hilbert-Kunz density function (denoted $\operatorname{HKd}(R, I)$) is a new invariant introduced by Trivedi, to study the Hilbert-Kunz multiplicity. In this talk, we discuss the Hilbert-Kunz density function for a projective toric variety X with a very ample T-Cartier divisor D on X. We give a nice interpretation of the HK density function $\operatorname{HKd}(X, D)$ in terms of the inherent combinatorial structure arising from the very ample lattice polytope P_D associated to the variety.

We also show that, for $k \geq 1$, the function $\mathrm{HKd}(X, kD)$ can be replaced (up to lower order terms in k) by another compactly supported continuous function φ_{kD} which is 'linear in k'. The limiting asymptotic growth of the Hilbert-Kunz multiplicity of (X, D) relative to the usual multiplicity relates to φ_D via an integral formula. For toric pairs (X, D) of fixed dimension, the limit (renormalized) is minimum if and only if P_D 'tiles the space'. This represents joint work with Prof. V. Trivedi.

Prosenjit Das (Indian Institute of Space Science and Technology)

Time: 15:50–16:20
Title: *Rank and Rigidity of Locally Nilpotent Derivations of Affine Fibrations*
Abstract: Given a locally nilpotent derivation (LND) D, the rank and rigidity of D is defined only when D is an LND of a polynomial algebra. In this talk we shall discuss the concept of rank and rigidity of an LND of an affine fibration; and thereafter observe criterion for affine fibrations (especially $\mathbb{A}^2$-fibrations) having LNDs to be trivial.

This talk is based on an ongoing joint work with Swapnil A. Lokhande, IIIT Vadodara.

Rajiv Kumar Garg (IIT Bombay)

Time: 16:25–16:55
Title: *Extremal Rays of Betti Cones*
Abstract: In 2009, Eisenbud and Schreyer prove that extremal rays of Betti cone over a polynomial ring are spanned by Betti diagrams of pure Cohen-Macaulay R-modules. We discuss extremal rays of Betti cone over a standard graded k-algebra and their purity. In particular, we show that Koszulness is a necessary condition for the purity of extremal rays.

Ganesh Kadu (University of Pune)

Time: 17:00–17:30
Title: *Hilbert Polynomial Associated to a Derived Functor*
Abstract: Let A be a Noetherian Cohen-Macaulay local ring of dimension $d \geq 1$. Let M be a finitely generated maximal Cohen-Macaulay A-module with $\dim M = d$ and I be an ideal of A. We investigate the following function

$$n \mapsto \ell(\mathrm{Tor}_i^A(M, A/I^{n+1}))$$

whenever the length is finite. It is well known that this numerical function is given by a polynomial for $n \gg 0$, denoted by $t_{M,i}^I(z)$ of degree at most $l(I) - 1$. We show that if I is $\mathfrak{m}$-primary with reduction number at most one then

$$t_{M,i}^I(n) = \ell(\mathrm{Tor}_i^R(M, R/I))\binom{n+d-1}{d-1}.$$

Moreover, we identify some cases in which the degree is maximal i.e. $l(I) - 1$. For example, in the case $i = 1$, we obtain

$$e_1^I(R)\mu(M) - e_1^I(M) - e_1^I(\mathrm{Syz}_1^I(M)) = \ell(\mathrm{Tor}_i^R(M, R/I))$$

We also show that when A is a hypersurface and $d = 1$ and I is any non parameter $\mathfrak{m}$-primary ideal then $t_{m,1}^I(z) < d - 1$ if and only if M is free. Also, if A is analytically unramified Cohen-Macaulay ring of dimension 1 and M is a maximal Cohen-Macaulay module and $\mathfrak{I} = \{\overline{I^n}\}$ is the integral closure filtration then we obtain $\deg t_{M,i}^{\mathfrak{I}}(z) = d - 1$.